hep-th/0403015, KIAS-P04016

Classical Stability of Black Branes*

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Abstract

Classical stability behaviors of various static black brane backgrounds under small perturbations have been summarized briefly. They include cases of black strings in AdS_5 space, charged black p-brane solutions in the type II supergravity, and the BTZ black string in four-dimensions. The relationship between dynamical stability and local thermodynamic stability - the so-called Gubser-Mitra conjecture - has also been checked for those cases.

I. INTRODUCTION

In the four-dimensional spacetime, event horizons of non-spherical topology are forbidden for asymptotically flat stationary black holes. In higher dimensions, however, the topology could be either hyperspherical (S^{D-2}) or hypercylindrical ($S^n \times R^{D-2-n}$ or $S^n \times S^{D-2-n}$). The four-dimensional Schwarzschild black hole in Einstein gravity is known to be stable classically under linearized perturbations. Recently, Ishibashi and Kodama [1] have shown that this stable behavior extends to hold for higher dimensional cases as well. Black hole solutions in higher dimensions having hypercylindrical horizons are called black strings or branes. Gregory and Laflamme [2] have investigated the stability of a black p-brane that is a product of the $(D-p)$-dimensional Schwarzschild black hole with the p-dimensional flat space, and found that such background is unstable as the compactification scale of extended directions becomes larger than the order of the horizon radius - the so-called Gregory-Laflamme instability. Gregory and Laflamme [3] also considered a class of magnetically charged black p-brane solutions for a stringy action containing the NS5-brane of the type II supergravity. For horizons with infinite extent, they have shown that the instability persists to appear but decreases as the charge increases to the extremal value. On the other hand, branes with extremal charge turned out to be stable [4]. Since their discovery of such linearized instability, black strings or branes have been believed to be generically unstable classically under small perturbations except for the cases of extremal or suitably compactified

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*Based on a talk presented at the 6th International Conference on Gravitation and Astrophysics held at Ewha Womans University, Seoul in Korea during Oct. 6-9, 2003.

ones, and the Gregory-Laflamme instability has been used to understand physical behaviors of various systems involving black brane configurations as in string theory.

In the context of string theory, however, black branes that Gregory and Laflamme considered are those having magnetic charges with respect to Neveu-Schwarz gauge fields only. Recently a wider class of black string or brane backgrounds has been studied in order to see whether or not the stability behavior drastically changes. In this talk I briefly summarize some of interesting results obtained so far, and report some new results for black $D3$-branes.

II. LINEARIZED STABILITY BEHAVIORS

In order to check the classical stability of a given black string or brane background under small perturbations, we seek any unstable linearized solution that grows in time and is regular spatially outside the event horizon. In the viewpoint of the Kaluza-Klein (KK) dimensional reduction, such unstable solution can be expanded in terms of KK modes along the extra directions characterized by the KK mass parameter m. In particular, s-wave perturbations that are spherically symmetric in the submanifold perpendicular to the spatial worldvolume are believed to be the strongest instability for most cases. The existence of such s-wave unstable mode can be checked by finding the so-called threshold mode that is static and the onset of instability. The number of unknown functions in the analysis can be further reduced by suitably choosing gauge conditions allowed in the system. If the set of coupled linearized equations allows any threshold mode solution with a non-vanishing threshold KK mass m^* for certain parameter values characterizing the background fields, the corresponding black brane is unstable.

A. Black strings in AdS space

In five-dimensional AdS space with or without a uniform 3-brane, one may have three types of static black string solutions characterized by a four-dimensional cosmological constant $\Lambda_4 = \pm 3/l_4^2$ and a parameter r_0 that is related to the mass density. They are Schwarzschild ($\Lambda_4 = 0$), dS$_4$-Schwarzschild ($\Lambda_4 > 0$), and AdS$_4$-Schwarzschild ($\Lambda_4 < 0$) black strings. As shown in Fig. 1 for varying r_0 with a given value of Λ_4 [5], there always exist non-vanishing threshold masses for cases of Schwarzschild [6] and dS$_4$-Schwarzschild black strings, implying instability as usual. For the case of AdS$_4$-Schwarzschild black strings, however, the threshold mass becomes smaller than the lowest allowed KK mass ($m_{\rm min}$) at a certain value of $r_0(\simeq 2.1)$. Without a 3-brane $m_{\rm min} = 4/l_4 = 0.4$ and this critical value of r_0 corresponds to the horizon radius of $r_+^{\rm cr} \simeq 0.20 l_4$. Thus, there is indeed no threshold mode solution for $r_+ \geq r_+^{\rm cr}$. In other words, the s-wave instability present in small size AdS$_4$-Schwarzschild black strings disappears as the horizon radius becomes larger than the order of the AdS$_4$ radius.

B. Black branes in type II string theory

Recently, Hirayama, Kang, and Lee [7] have analyzed the linearized stability of a wider class of magnetically charged black p-brane solutions for the string gravity action given by

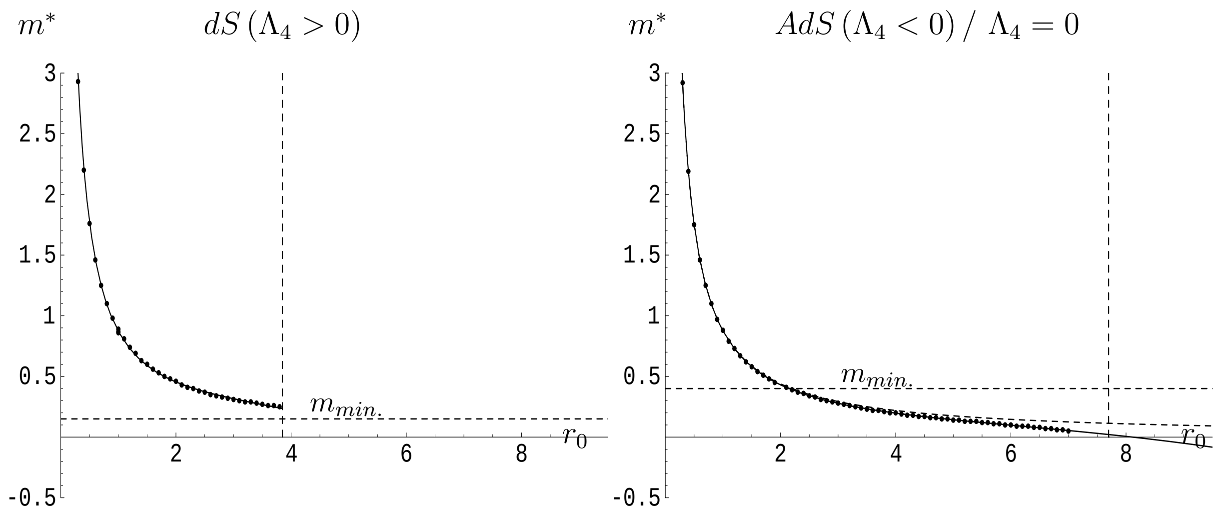

FIG. 1. The lefthand side: threshold masses for varying r_0 with given $l_4 = 10$ in the dS_4-Schwarzschild black strings. The vertical dotted line denotes the maximum possible value (Nariai limit), $r_0 \simeq 3.85$. Note that all threshold masses are larger than the lowest value in the KK mass spectrum (*i.e.*, $m^*(r_0 \simeq 3.85) = 0.29 > m_{\rm min.} \simeq 0.15$). The righthand side: For the case of Schwarzschild black strings threshold masses (dotted curve) never cross the horizontal axis ($m_{\rm min} = 0$). For AdS_4-Schwarzschild black strings, however, the threshold mass becomes smaller than the lowest allowed KK mass at a certain value of $r_0(\simeq 2.1)$, implying no unstable solution for r_0 larger than such critical value.

$$\mathrm{I} = \int d^D x \sqrt{-g}\left[R - \frac{1}{2}(\partial\phi)^2 - \frac{1}{2n!}e^{a\phi}F_n^2\right] \tag{1}$$

in the Einstein frame. It turns out that the stability of these black p-branes behaves very differently depending on the coupling parameter a. That is, there exists a critical value of the coupling parameter $a_{\rm cr}(D,p) = (D-3-p)/\sqrt{(D-2)/2}$ determined by the full spacetime dimension D and the dimension of the spatial worldvolume $p = D-2-n$. The case that Gregory and Laflamme studied is precisely when $a = a_{\rm cr}$. In this case black branes with horizons of infinite extent are always unstable as explained above, and magnetically charged NS5-branes of the type II supergravity belong to this class. When $0 \leq a < a_{\rm cr}$, black branes with small charge are unstable as usual. As the charge increases, however, the instability decreases and eventually disappears at a certain critical value of the charge density which could be even far from the extremal point. Magnetically charged black D0, F1, D1, D2, D4 branes of the type II string theory belong to this class for instance. When $a > a_{\rm cr}$, on the other hand, the instability persists all the way down to the extremal point. Interestingly it should be noticed that in this case the threshold mass starts to increase again near the extremal point ($q \simeq 1$) as can be seen in Fig. 2. Such stability behavior in the presence of charge never has been expected in the literature. Magnetically charged black D5 and D6 branes in the type II supergravity are in this case for example. However it is shown that all black branes mentioned above are stable at the extremal point, which might be expected due to the BPS nature of extremal solutions in string theory. On the lefthand side of Fig. 2 behaviors of threshold masses are illustrated for some black 4-branes as the charge increases.

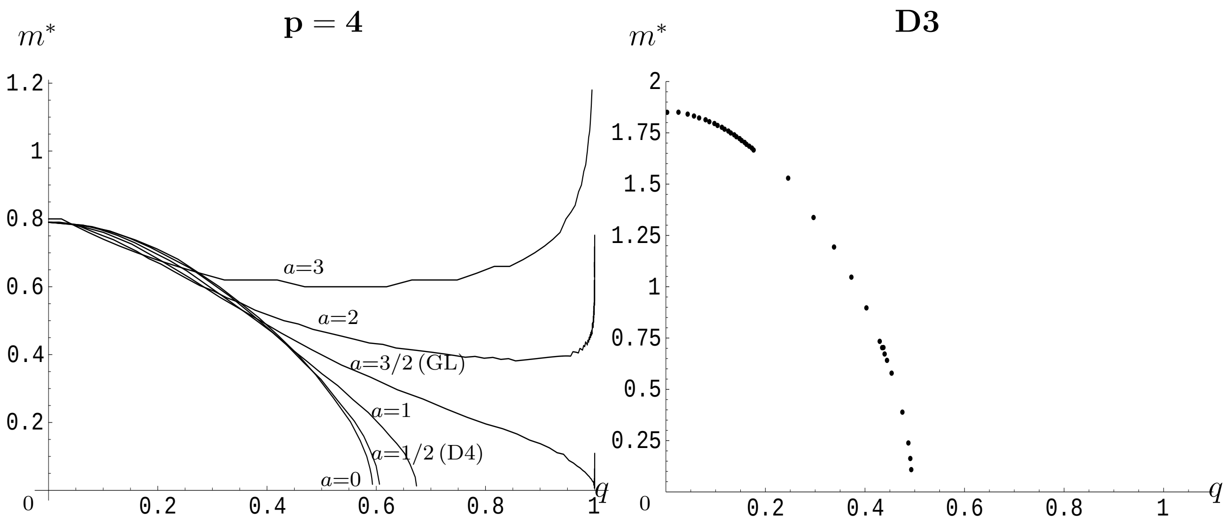

FIG. 2. The lefthand side: behavior of threshold masses for black 4-branes in $D = 10$ at various values of a with fixed mass density $M = 2^5$. $m^* \simeq 1.581/r_H \simeq 0.791$ with $r_H = 2$ for uncharged black branes (*i.e.*, $q = 0$). The extremality parameter $q = 1$ for extremal branes. Critical values at which the instability disappears are $q_{\rm cr} \simeq 0.593$, 0.606, 0.674 for cases of $a = 0$, $1/2$, 1, respectively. The righthand side: behavior of threshold masses for black $D3$-branes with mass density $M = 5$. The critical value is $q_{\rm cr} \simeq 0.49$.

C. Black $D3$-branes

The case of $n = D/2$ and $a = 0$ with a self-dual n-form field strength $\mathbf{F} = {}^*\mathbf{F}$ in Eq. (1) is treated separately for some technical reasons. This case includes black D3-branes (*i.e.*, $n = 5$) in the type II supergravity for which the AdS/CFT correspondence has been understood very well in string theory. In contrast to previous cases mentioned above, the fluctuation of the dilaton field is completely decoupled and can be set to be zero, but the s-wave perturbation of the field strength should not be frozen in order to be consistent with the metric perturbation as a black brane gets charged. As shown in the numerical results on the righthand side of Fig. 2 for black $D3$-branes, a black brane in this class is unstable when it has small charge density. As the charge density increases for given mass density, however, the instability decreases down to zero at a certain finite value of the charge density, and then the black brane becomes stable all the way down to the extremal point [8,9].

D. Black strings in $D \leq 4$

It is possible to have stationary black string or brane solutions even in four dimensions when a negative cosmological constant is present. Interestingly it is likely that all known stationary black branes in four or three dimensions are stable since they are thermodynamically stable. In particular, the case of BTZ black strings has been explicitly checked to be stable by linearized analysis for any type of perturbations [10].

III. THERMODYNAMIC STABILITY BEHAVIOR

One of naive explanations for the occurrence of the Gregory-Laflamme instability is that for a given black string or brane configuration there exists some other black hole configuration such as a hyperspherical black hole that has the same mass and charge, but possesses larger entropy [2,3]. Recently, Gubser and Mitra [11] refined such entropy comparison argument, and proposed a conjecture that a black brane with a non-compact translational symmetry is classically stable if and only if it is locally thermodynamically stable. The proof of this Gubser-Mitra (GM) conjecture has been argued for a certain class of black brane systems by Reall [12]. This interesting relationship between the classical dynamical stability and the local thermodynamic stability has been explicitly checked to hold for various black string or brane systems [13,11,10,7,9,8]. When the translational symmetry along the horizon is broken, one can see some disagreements for onset points of instability as shown in the stability analysis for AdS_4-Schwarzschild black strings in AdS space [5]. It also should be pointed out that the GM conjecture simply gives the information about when a black string or brane becomes stable or unstable. It does not explain or predict other details of classical stability behaviors [7].

IV. DISCUSSION

To conclude, it is briefly summarized how the classical linearized stability behaves for various black string or brane solutions in several classes of theories. In lower dimensions (*i.e.*, $D \leq 4$) all black branes seem to be stable. In higher dimensions (*i.e.*, $D > 4$) all neutral black branes considered are unstable unless they are suitably compactified or have some suitable AdS nature. As a black brane gets charge, the s-wave instability could either persist all the way down to the extremal point or disappear at a certain value of charge density, depending on its theory. For all cases we considered, even if a non-extremal black brane possesses the s-wave instability at any value of the charge density, its extremal one turns out to be free of such instability. On the other hand, the GM conjecture turned out to hold well for the cases of having the translation symmetry along the spatial worldvolume.

Further work is required to have deeper understandings of these diverse stability behaviors. Some of interesting open problems are

- Physical understanding of the transition point in the parameter space at which the stability behavior changes.
- The consequence of the Gregory-Laflamme instability and its evolution afterwards when it is present.
- General proof of the stability when the s-wave instability is absent.
- Possibility of some simple criterion for the stability such as the GM conjecture in the case of stationary non-uniform black branes.

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